A Refinement Operator Based Learning Algorithm for the $\mathcal{ALC}$ Description Logic

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Abstract With the advent of the Semantic Web, description logics have become one of the most prominent paradigms for knowledge representation and reasoning. Progress in research and applications, however, faces a bottleneck due to the lack of available knowledge bases, and it is paramount that suitable automated methods for their acquisition will be developed. In this paper, we provide the first learning algorithm based on refinement operators for the most fundamental description logic $\mathcal{ALC}$. We develop the algorithm from thorough theoretical foundations and report on a prototype implementation.

1 Introduction

The Semantic Web is gaining momentum. Semantic Technologies, based on the same underlying principles, are being applied in adjacent areas such as Software Engineering and Content Management, and industrial interest is rising rapidly. Fundamental to these approaches is the modelling of knowledge by means of ontologies, and the single most popular paradigm for this is by using the Web Ontology Language OWL,[3] which has been recommended by the World Wide Web Consortium (W3C) since 2004.

Progress in research and applications, however, faces a bottleneck due to the lack of available OWL knowledge bases. Considerable effort is therefore currently being invested into developing automated means for the acquisition of ontologies. Most of the currently pursued approaches, however, neglect the expressive power of OWL and are only capable of learning inexpressive ontologies, such as taxonomic hierarchies. As such, they fail by far in leveraging the potential inherent in the expressive features of the Web Ontology Language.

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[3] http://www.w3.org/2004/OWL

From a logical perspective, OWL is basically an expressive description logic (DL) [1]. It is therefore natural to attempt an adaptation of logic-based approaches to machine-learning for automated ontology acquisition. Inspired by the success of Inductive Logic Programming (ILP), we pursue the transfer of ILP methods [13] to DLs, and in this paper we report on a resulting learning algorithm. Our approach is based on a thorough theoretical analysis of the potential and limitations of refinement operators for DLs. We make the following contributions:

1. development of a refinement operator, which conforms to theoretical findings
2. design of an algorithm handling the unavoidable limitations of this operator
3. provision of a preliminary evaluation

The algorithm was created with extensibility to additional (non-$\mathcal{ALC}$) concept constructors, e.g. number restrictions, in mind. In contrast to previous approaches [6,7,8], we pay more attention to finding simple, non-overfitting solutions of the learning problem.

The paper is structured as follows. After some preliminaries in Section 2, we introduce our refinement operator in Section 3 and show that it conforms to the desired theoretical properties. In Section 4 we extend this refinement operator to a learning algorithm. In Section 5, we report on our prototype implementation and preliminary evaluation. We discuss related work in Section 6 and conclude in Section 7. Proofs had to be omitted for lack of space, but can be found in the technical report [10].

2 Preliminaries

2.1 Description Logics

Description logics represent knowledge in terms of *objects*, *concepts*, and *roles*. Objects correspond to constants, concepts to unary predicates, and roles to binary predicates in first order logic. In DL systems information is stored in a *knowledge base*, which is a set of axioms. It is divided in (at least) two parts: *TBox* (terminology) and *ABox* (assertions). The ABox contains *assertions* about objects. It relates objects to concepts and roles. The TBox describes the *terminology* by relating concepts and roles.

We briefly introduce the $\mathcal{ALC}$ description logic, which is the target language of our learning algorithm and refer to [1] for further background on description logics. As usual in logics, interpretations are used to assign a meaning to syntactic constructs. Let N_I denote the set of objects, N_C denote the set of atomic concepts, and N_R denote the set of roles. An *interpretation* $\mathcal{I}$ consists of a non-empty *interpretation domain* $\Delta^\mathcal{I}$ and an *interpretation function* $\cdot^\mathcal{I}$, which assigns to each object $a \in N_I$ an element of $\Delta^\mathcal{I}$, to each concept $A \in N_C$ a set $A^\mathcal{I} \subseteq \Delta^\mathcal{I}$, and to each role $r \in N_R$ a binary relation $r^\mathcal{I} \subseteq \Delta^\mathcal{I} \times \Delta^\mathcal{I}$. Interpretations are extended to concepts as shown in Table 1, and to other elements of a knowledge base in a straightforward way. An interpretation, which satisfies an

construct	syntax	semantics
atomic concept	A	$A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$
role	r	$r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
top	$\top$	$\Delta^{\mathcal{I}}$
bottom	$\bot$	$\emptyset$
conjunction	$C \sqcap D$	$(C \sqcap D)^{\mathcal{I}} = C^{\mathcal{I}} \cap D^{\mathcal{I}}$
disjunction	$C \sqcup D$	$(C \sqcup D)^{\mathcal{I}} = C^{\mathcal{I}} \cup D^{\mathcal{I}}$
negation	$\neg C$	$(\neg C)^{\mathcal{I}} = \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$
existential	$\exists r.C$	$(\exists r.C)^{\mathcal{I}} = \{a \mid$ $\exists b.(a,b) \in r^{\mathcal{I}}$ and $b \in C^{\mathcal{I}}\}$
universal	$\forall r.C$	$(\forall r.C)^{\mathcal{I}} = \{a \mid$ $\forall b.(a,b) \in r^{\mathcal{I}}$ implies $b \in C^{\mathcal{I}}\}$

Table 1. $\mathcal{ALC}$ syntax and semantics

axiom (set of axioms) is called a model of this axiom (set of axioms). An $\mathcal{ALC}$ concept is in *negation normal form* if negation only occurs in front of concept names.

If C and D are concepts, then $C \sqsubseteq D$ and $C \equiv D$ are *terminological axioms*. The former axioms are called *inclusions* and the latter *equalities*. An equality whose left hand side is an atomic concept is a *concept definition*.

It is the aim of *inference algorithms* to extract implicit knowledge from a given knowledge base. Standard reasoning tasks include *instance checks*, *retrieval* and *subsumption*. We will only explicitly define the latter. Let C, D be concepts and $\mathcal{T}$ a TBox. C *is subsumed by* D, denoted by $C \sqsubseteq D$, iff for any interpretation $\mathcal{I}$ we have $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$. C *is subsumed by* D *with respect to* $\mathcal{T}$ (denoted by $C \sqsubseteq_{\mathcal{T}} D$) iff for any model $\mathcal{I}$ of $\mathcal{T}$ we have $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$. C *is equivalent to* D *(with respect to* $\mathcal{T}$*)*, denoted by $C \equiv D$ ($C \equiv_{\mathcal{T}} D$), iff $C \sqsubseteq D$ ($C \sqsubseteq_{\mathcal{T}} D$) and $D \sqsubseteq C$ ($D \sqsubseteq_{\mathcal{T}} C$). C *is strictly subsumed by* D *(with respect to* $\mathcal{T}$*)*, denoted by $C \sqsubset D$ ($C \sqsubset_{\mathcal{T}} D$), iff $C \sqsubseteq D$ ($C \sqsubseteq_{\mathcal{T}} D$) and not $C \equiv D$ ($C \equiv_{\mathcal{T}} D$).

2.2 Learning in Description Logics using Refinement Operators

Definition 1 (learning problem in description logics). *Let a concept name* `Target`*, a knowledge base* $\mathcal{K}$ *(not containing* `Target`*), and sets* E^+ *and* E^- *with elements of the form* `Target`(a) $(a \in N_I)$ *be given. The learning problem is to find a concept* C *such that* `Target` *does not occur in* C *and for* $\mathcal{K}' = \mathcal{K} \cup \{\texttt{Target} \equiv C\}$ *we have* $\mathcal{K}' \models E^+$ *and* $\mathcal{K}' \not\models E^-$.

By Occam's razor [3] simple solutions of the learning problem are to be preferred over more complex ones, because they have a higher predictive quality. We measure simplicity as the *length* of a concept, which is defined in a straightforward way, namely as the sum of the numbers of concept, role, quantifier, and connective symbols occurring in the concept.

The goal of learning is to find a correct concept with respect to the examples. This can be seen as a search process in the space of concepts. A natural idea is to impose an ordering on this search space and use operators to traverse it, which is the purpose of *refinement operators*. Intuitively, downward (upward) refinement operators construct specialisations (generalisations) of hypotheses.

A *quasi-ordering* is a reflexive and transitive relation. Let S be a set and $\preceq$ a quasi-ordering on S. In the quasi-ordered space $(S, \preceq)$ a *downward (upward) refinement operator* ρ is a mapping from S to 2^S, such that for any $C \in S$ we have that $C' \in \rho(C)$ implies $C' \preceq C$ $(C \preceq C')$. C' is called a *specialisation* (*generalisation*) of C. Quasi-orderings can be used for searching in the space of concepts. As ordering we can use subsumption. If a concept C subsumes a concept D $(D \sqsubseteq C)$, then C covers all examples, which are covered by D, which makes subsumption a suitable order.

Definition 2. *A refinement operator in the quasi-ordered space* $(\mathcal{ALC}, \sqsubseteq_{\mathcal{T}})$ *is called an* $\mathcal{ALC}$ refinement operator*.*

We need to introduce some notions for refinement operators. A *refinement chain of an* $\mathcal{ALC}$ *refinement operator* ρ *of length* n from a concept C to a concept D is a finite sequence $C_0, C_1, \ldots, C_n$ of concepts, such that $C = C_0, C_1 \in \rho(C_0), C_2 \in \rho(C_1), \ldots, C_n \in \rho(C_{n-1}), D = C_n$. This refinement chain *goes through* E iff there is an i $(1 \leq i \leq n)$ such that $E = C_i$. We say that D can be reached from C by ρ if there exists a refinement chain from C to D. $\rho^*(C)$ denotes the set of all concepts, which can be reached from C by ρ. $\rho^m(C)$ denotes the set of all concepts, which can be reached from C by a refinement chain of ρ of length m. If we look at refinements of an operator ρ, we will often write $C \rightsquigarrow_\rho D$ instead of $D \in \rho(C)$. If the used operator is clear from the context it is usually omitted, i.e. we write $C \rightsquigarrow D$.

We will introduce the concept of weak equality of concepts, which is similar to equality of concepts, but takes into account that the order of elements in conjunctions and disjunctions is not important. We say that the concepts C and D are *weakly (syntactically) equal*, denoted by $C \simeq D$ iff they are equal up to permutation of arguments of conjunction and disjunction. Two sets S_1 and S_2 of concepts are weakly equal if for any $C_1 \in S_1$ there is a $C'_1 \in S_2$ such that $C_1 \simeq C'_1$ and vice versa. Weak equality of concepts is coarser than equality and finer than equivalence (viewing the equivalence, equality, and weak equality of concepts as equivalence classes). Refinement operators can have certain properties, which can be used to evaluate their usefulness for learning hypothesis.

Definition 3. *An* $\mathcal{ALC}$ *refinement operator* ρ *is called*

- (locally) finite *iff* $\rho(C)$ *is finite for any concept* C*.*
- redundant *iff there exists a refinement chain from a concept* C *to a concept* D*, which does not go through some concept* E *and a refinement chain from* C *to a concept weakly equal to* D*, which does go through* E*.*
- proper *iff for all concepts* C *and* D*,* $D \in \rho(C)$ *implies* $C \not\equiv D$*.*
- ideal *iff it is finite, complete (see below), and proper.*

An $\mathcal{ALC}$ *downward refinement operator* ρ *is called*

– complete *iff for all concepts* C, D *with* $C \sqsubset_{\mathcal{T}} D$ *we can reach a concept* E *with* $E \equiv C$ *from* D *by* ρ.
– weakly complete *iff for all concepts* $C \sqsubset_{\mathcal{T}} \top$ *we can reach a concept* E *with* $E \equiv C$ *from* $\top$ *by* ρ.

The corresponding notions for upward refinement operators are defined dually.

3 Designing a Refinement Operator

To design a suitable operator, we first look at theoretical limitations. The following theorem from [9] provides a full analysis of the properties of $\mathcal{ALC}$ refinement operators:

Theorem 1 (Property Theorem). *Considering the properties completeness, weak completeness, properness, finiteness, and non-redundancy the following are maximal sets of properties (in the sense that no other of the mentioned properties can be added) of* $\mathcal{ALC}$ *refinement operators (see [9] for details):*

1. {*weakly complete*, *complete*, *finite*}
2. {*weakly complete*, *complete*, *proper*}
3. {*weakly complete*, *non-redundant*, *finite*}
4. {*weakly complete*, *non-redundant*, *proper*}
5. {*non-redundant*, *finite*, *proper*}

Incomplete operators are not interesting, because we may then be unable to find possible solutions, so we can ignore the fifth property combination. We can see from the other combinations, that we can have either finity or properness as a property of an $\mathcal{ALC}$ refinement operator – but not both at the same time. Since we are able to handle infinity quite well, as we will describe in Section 4, we will aim for properness. Our learning algorithm will perform a top-down search, so the fourth combination seems to be desirable, because weak completeness is sufficient in this case. However, an incomplete, but weakly complete operator cannot support some of the features which we consider essential in our learning algorithm. Hence, we decided to use the second combination.

We proceed as follows: First, we define a refinement operator and prove its completeness. We then extend it to a complete and proper operator. Section 4 will show how we handle the problems of redundancy and infinity in the learning algorithm.

For each $A \in N_C$, we define $\text{nb}_\downarrow(A) = \{A' \mid A' \in N_C, \text{ there is no } A'' \in N_C \text{ with } A' \sqsubset_{\mathcal{T}} A'' \sqsubset_{\mathcal{T}} A\}$. $\text{nb}_\uparrow(A)$ is defined analogously. In the sequel, we will analyse the refinement operator $\rho_\downarrow$ given by:

$$\rho_\downarrow(C) = \begin{cases} \{\bot\} \cup \rho'_\downarrow(C) & \text{if } C = \top \\ \rho'_\downarrow(C) & \text{otherwise} \end{cases}$$

where the operator $\rho'_\downarrow$ is defined as in Figure 1. The definition refers to a set M which is inductively defined as follows: All elements in $\{A \mid A \in N_C, \text{nb}_\uparrow(A) = \emptyset\}$

(= most general atomic concepts), $\{\neg A \mid A \in N_C, \mathrm{nb}_\downarrow(A) = \emptyset\}$ (= negated most specific atomic concepts), and $\{\exists r.\top \mid r \in N_R\}$ are in M. If a concept C is in M, then $\forall r.C$ with $r \in N_R$ is also in M.

$$\rho'_\downarrow(C) = \begin{cases} \emptyset & \text{if } C = \bot \\ \{C_1 \sqcup \cdots \sqcup C_n \mid C_i \in M \ (1 \leq i \leq n)\} & \text{if } C = \top \\ \{A' \mid A' \in \mathrm{nb}_\downarrow(A)\} \cup \{A \sqcap D \mid D \in \rho'_\downarrow(\top)\} & \text{if } C = A \ (A \in N_C) \\ \{\neg A' \mid A' \in \mathrm{nb}_\uparrow(A)\} \cup \{\neg A \sqcap D \mid D \in \rho'_\downarrow(\top)\} & \text{if } C = \neg A \ (A \in N_C) \\ \{\exists r.E \mid E \in \rho'_\downarrow(D)\} \cup \ \{\exists r.D \sqcap E \mid E \in \rho'_\downarrow(\top)\} & \text{if } C = \exists r.D \\ \{\forall r.E \mid E \in \rho'_\downarrow(D)\} \cup \ \{\forall r.D \sqcap E \mid E \in \rho'_\downarrow(\top)\} & \text{if } C = \forall r.D \\ \quad \cup \{\forall r.\bot \mid D = A \in N_C \text{ and } \mathrm{nb}_\downarrow(A) = \emptyset\} & \\ \{C_1 \sqcap \cdots \sqcap C_{i-1} \sqcap D \sqcap C_{i+1} \sqcap \cdots \sqcap C_n \mid & \text{if } C = C_1 \sqcap \cdots \sqcap C_n \\ \quad D \in \rho'_\downarrow(C_i), 1 \leq i \leq n\} & (n \geq 2) \\ \{C_1 \sqcup \cdots \sqcup C_{i-1} \sqcup D \sqcup C_{i+1} \sqcup \cdots \sqcup C_n \mid & \text{if } C = C_1 \sqcup \cdots \sqcup C_n \\ \quad D \in \rho'_\downarrow(C_i), 1 \leq i \leq n\} & (n \geq 2) \\ \quad \cup \{(C_1 \sqcup \cdots \sqcup C_n) \sqcap D \mid D \in \rho'_\downarrow(\top)\} & \end{cases}$$

Figure 1. definition of $\rho'_\downarrow$

Proposition 1. *$\rho_\downarrow$ is an $\mathcal{ALC}$ downward refinement operator.*

Proof. We have to show that $D \in \rho_\downarrow(C)$ implies $D \sqsubseteq_\mathcal{T} C$. We show $D \sqsubseteq C$, i.e. consider an empty TBox. Since $D \sqsubseteq C$ implies $D \sqsubseteq_\mathcal{T} C$ by the definition of subsumption, the desired result follows.

We do this by structural induction of $\mathcal{ALC}$ concepts in negation normal form. Obviously in all cases where $D = C \sqcap C'$, i.e. C is extended conjunctively by a concept C' we have $D \sqsubseteq C$, so these cases can be ignored.

- $C = \bot$: $D \in \rho_\downarrow(C)$ is impossible, because $\rho_\downarrow(\bot) = \emptyset$.
- $C = \top$: $D \sqsubseteq C$ is trivially true.
- $C = A$ ($A \in N_C$): $D \in \rho_\downarrow(C)$ implies that D is also an atomic concept and $D \sqsubset_\mathcal{T} C$. Thus $D \sqsubseteq C$.
- $C = \neg A$: $D \in \rho_\downarrow(C)$ implies that D is of the form $\neg A'$ with $A \sqsubset_\mathcal{T} A'$. Thus $D \sqsubseteq C$. $A \sqsubset_\mathcal{T} A'$ implies $\neg A' \sqsubset_\mathcal{T} \neg A$ by the semantics of negation.
- $C = \exists r.C'$: $D \in \rho_\downarrow(C)$ implies that D is of the form $\exists r.D'$. We have $D' \sqsubseteq C'$ by induction. For existential restrictions $\exists r.E \sqsubseteq \exists r.E'$ if $E \sqsubset E'$ holds in general [2]. Thus we also have $\exists r.D' \sqsubseteq \exists r.C'$.
- $C = \forall r.C'$: This case is analogous to the previous one. For universal restrictions $\forall r.E \sqsubseteq \forall r.E'$ if $E \sqsubset_\mathcal{T} E'$ holds in general [2].
- $C = C_1 \sqcap \cdots \sqcap C_n$: In this case one element of the conjunction is refined, so $D \sqsubseteq C$ follows by induction.

- $C = C_1 \sqcup \cdots \sqcup C_n$: In this case one element of the disjunction is refined, so $D \sqsubseteq C$ follows by induction.

A distinguishing feature of $\rho_\downarrow$ compared to other refinement operators for learning concepts in DLs [2,6] is that it makes use of the subsumption hierarchy. This is useful, since the operator can make use of knowledge contained implicitly in the TBox. Note that $\rho_\downarrow$ is infinite. The reason is that the set M is infinite and, furthermore, we put no boundary on the number of elements in the disjunctions, which are refinements of the top concept.

3.1 Completeness of the Operator

To investigate the completeness of the operator, we define a set $S_\downarrow$ of $\mathcal{ALC}$ concepts in negation normal form as follows:

Definition 4 **($S_\downarrow$).** *We define $S_\downarrow = S'_\downarrow \cup \{\bot\}$, where $S'_\downarrow$ is defined as follows:*

1. *If $A \in N_C$ then $A \in S'_\downarrow$ and $\neg A \in S'_\downarrow$.*
2. *If $r \in N_R$ then $\forall r.\bot \in S'_\downarrow$, $\exists r.\top \in S'_\downarrow$.*
3. *If $C, C_1, \dots, C_m$ are in $S'_\downarrow$ then the following concepts are also in $S'_\downarrow$:*
   - *$\exists r.C$, $\forall r.C$, $C_1 \sqcap \cdots \sqcap C_m$, and*
   - *$C_1 \sqcup \cdots \sqcup C_m$ if for all i $(1 \le i \le m)$ C_i is not of the form $D_1 \sqcap \cdots \sqcap D_n$ where all D_j $(1 \le j \le n)$ are of the form $E_1 \sqcup \cdots \sqcup E_p$.*

In $S_\downarrow$, we do not use the $\top$ and $\bot$ symbols directly and we make a restriction on disjunctions, i.e. we do not allow that elements of a disjunction are conjunctions, which in turn only consist of disjunctions. It can be shown that for any $\mathcal{ALC}$ concept C there exists a concept $D \in S_\downarrow$ such that $D \equiv C$.

Lemma 1 **($S_\downarrow$).** *For any $\mathcal{ALC}$ concept C there exists a concept $D \in S_\downarrow$ such that $D \equiv C$.*

Proof. We first transform C to negation normal form. The proof consists of three steps: First we will eliminate $\top$ symbols unless they occur in existential restrictions (because in Definition 4 $\exists r.\top$ is used in the induction base opposed to using $\top$ directly). After that, we do something similar with the bottom symbol. In a third step we will eliminate disjunctions violating the criterion in Definition 4. After these three steps, we obviously obtain a concept, which is in $S_\downarrow$.

We eliminate $\top$-symbols by applying the following rewrite rules:

$$
\begin{aligned}
C_1 \sqcap \cdots \sqcap C_{i-1} \sqcap \top \sqcap C_{i+1} \sqcap \cdots \sqcap C_n &\rightarrow C_1 \sqcap \cdots \sqcap C_{i-1} \sqcap C_{i+1} \sqcap \cdots \sqcap C_n \\
C_1 \sqcup \cdots \sqcup C_{i-1} \sqcup \top \sqcup C_{i+1} \sqcup \cdots \sqcup C_n &\rightarrow \top \\
\forall r.\top &\rightarrow \top
\end{aligned}
$$

Obviously, these $\top$-elimination steps preserve equivalence. We exhaustively apply these steps (since every step reduces the length of the concept there can be only finitely many such steps) to get a concept C'. Note that $C' \neq \top$ (otherwise

$C' \equiv C \equiv \top$) and in C' the top concept only appears in existential restrictions, i.e. in the form $\exists r.\top$.

$\bot$ symbols are eliminated by the following rewrite rules:

$$\begin{aligned} C_1 \sqcap \dots \sqcap C_{i-1} \sqcap \bot \sqcap C_{i+1} \sqcap \dots \sqcap C_n &\rightarrow \bot \\ C_1 \sqcup \dots \sqcup C_{i-1} \sqcup \bot \sqcup C_{i+1} \sqcup \dots \sqcup C_n &\rightarrow C_1 \sqcup \dots \sqcup C_{i-1} \sqcup C_{i+1} \sqcup \dots \sqcup C_n \\ \exists r.\bot &\rightarrow \bot \end{aligned}$$

These steps also preserve equivalence. After exhaustively applying these steps we either get the $\bot$ symbol itself (which is in $S_\downarrow$) or the $\bot$ symbol only appears in universal restrictions, i.e. in the form $\forall r.\bot$.

Next we have to eliminate disjunctions, which do not satisfy Definition 4. Say we have such a disjunction $C_1 \sqcup \dots \sqcup C_m$. Then there is a C_i $(1 \leq i \leq m)$, which is a conjunction consisting only of disjunctions. Without loss of generality we assume $i = 1$ (the order of elements in a disjunction is not important), i.e. we can write $C_1 = D_1 \sqcap \dots \sqcap D_n$ and $D_1 = E_1 \sqcup \dots \sqcup E_p$. This means we can apply the following equivalence preserving rewriting rule:

$$\begin{aligned} ((E_1 \sqcup \dots \sqcup E_p) \sqcap D_2 \sqcap \dots \sqcap D_n) \sqcup C_2 \sqcup \dots \sqcup C_m &\rightarrow \\ (E_1 \sqcap D_2 \sqcap \dots \sqcap D_n) \sqcup \dots \sqcup (E_p \sqcap D_2 \sqcap \dots \sqcap D_n) \sqcup C_2 \dots \sqcup C_m \end{aligned}$$

Note that E_i $(1 \leq i \leq p)$ cannot be a disjunction. Let C_1' be the replacement of C_1 after applying the rewriting rule. Obviously, C_1' is no more a disjunction where an element is a conjunction of disjunctions (because any E_i is not a disjunction). If we apply this rule to all applicable C_i $(1 \leq i \leq m)$, then we obtain a concept C'' equivalent to $C_1 \sqcup \dots \sqcup C_m$, which is in $S_\downarrow$.

Hence, we have shown that we can construct a concept $C'' \equiv C' \equiv C$ with $C'' \in S_\downarrow$, which completes the proof.

This allows us to show weak completeness by proving that every element in $S_\downarrow$ can be reached from $\top$ by $\rho_\downarrow$.

Proposition 2 (weak completeness of $\rho_\downarrow$). *$\rho_\downarrow$ is weakly complete.*

Proof. We have to show that for any concept C with $C \sqsubset_\mathcal{T} \top$ a concept E with $E \equiv C$ can be reached from $\top$ by $\rho_\downarrow$. Due to Lemma 1, it is sufficient to show that all concepts in $S_\downarrow$ can be reached from $\top$ by $\rho_\downarrow$.

We claim that $\rho_\downarrow^*(\top) = S_\downarrow$ and prove this by induction over the structure of concepts in $S_\downarrow$ (see Definition 4). The bottom concept itself can be reached by a one-step refinement of the $\top$ symbol, so we just have to analyse the elements in $S'_\downarrow$.

– Induction Base: An atomic concept A can be reached from $\top$ by a refinement chain of the following form:

$$\top \overset{\top}{\rightsquigarrow} A_1 \overset{A}{\rightsquigarrow} \dots \overset{A}{\rightsquigarrow} A_n \overset{A}{\rightsquigarrow} A$$

The operator descends the subsumption hierarchy and reaches A in a finite number of steps (there are only finitely many atomic concepts). Negated atomic concepts can be handled analogously.
$\forall r.\bot$ can also be reached by descending the subsumption hierarchy:

$$\top \overset{\top}{\rightsquigarrow} \forall r.A_1 \overset{A}{\rightsquigarrow} \ldots \overset{A}{\rightsquigarrow} \forall r.A_n \overset{A}{\rightsquigarrow} \forall r.\bot$$

$\exists r.\top$ can be reached by a one step refinement from $\top$.

– Induction Step:
  - $\exists r.C$: We have $\top \overset{\top}{\rightsquigarrow} \exists r.\top$ and by induction we can reach C from $\top$ by $\rho_\downarrow$.
  - $\forall r.C$: We have $\forall r.C \in \rho_\downarrow^*(\top)$ if $C \in \rho_\downarrow^*(\top)$, which is true by induction.
  - $C_1 \sqcup \cdots \sqcup C_m$: We will look at the elements of the disjunction separately. We have to show that for any i we have $C_i \in \rho^*(m)$ for an $m \in M$ (where M is defined as in the definition of $\rho_\downarrow$). Without loss of generality we show this for C_1, i.e. $i = 1$. We do a case distinction based on the structure of C_1 (obviously C_1 is not a disjunction).
    * C_1 is an atomic concept A: In this case we pick a most general atomic concept $A_1 \in S$ and refine it:

    $$A_1 \overset{A}{\rightsquigarrow} \ldots \overset{A}{\rightsquigarrow} A_n \overset{A}{\rightsquigarrow} A$$

    * C_1 is a negated atomic concept $\neg A$: We pick a most special atomic concept $A_1 \in S$ and refine it:

    $$A_1 \overset{\neg A}{\rightsquigarrow} \ldots \overset{\neg A}{\rightsquigarrow} A_n \overset{\neg A}{\rightsquigarrow} A$$

    * $C_1 = \forall r.\bot$: We refine to $\forall r.A_1$, where A_1 is itself a refinement of the top concept. Then we can refine to $\forall r.\bot$:

    $$\forall r.A_1 \overset{A}{\rightsquigarrow} \ldots \overset{A}{\rightsquigarrow} \forall r.A_n \overset{A}{\rightsquigarrow} \forall r.\bot$$

    * $C_1 = \exists r.\top$: $\exists r.\top$ is in M.
    * $C_1 = \forall r.D$: By induction, $D \in \rho_\downarrow^*(\top)$, so there is a concept $\forall r.E_1 \in S$, which we can refine to $\forall r.D$:

    $$\forall r.E_1 \rightsquigarrow \ldots \rightsquigarrow \forall r.E_n \rightsquigarrow \forall r.D$$

    * $C_1 = \exists r.D$: We choose $\exists r.\top \in M$ and by induction $D \in \rho_\downarrow^*(\top)$.
    * $C_1 = D_1 \sqcap \cdots \sqcap D_n$: By Definition of $S_\downarrow$ (Definition 4), we know that there exists a j, such that D_j is not a disjunction (and obviously also not a conjunction). So there is one D_j of the form $\exists r.\top$, $\bot$, A, $\neg A$, $\exists r.C$ or $\forall r.C$. These can be produced exactly as we have shown in the previous cases. For any of these constructs $\rho_\downarrow$ allows to extend these constructs to a conjunction with elements in $\rho_\downarrow(\top)$. By induction, we know for all i ($1 \leq i \leq n$) that $D_i \in \rho_\downarrow^*(\top)$. So we can produce a concept weakly equal to C_1 by first creating D_j and then extending this to a conjunction by generating all $D_i's$ ($1 \leq i \leq n, i \neq j$).

- $C_1 \sqcap \dots \sqcap C_m$: By induction, we know $C_1 \in \rho^*(\top)$, so we first create C_1 and then add all other elements to the conjunction:

$$\top \overset{\top}{\rightsquigarrow} D_1 \dots \rightsquigarrow C_1 \overset{\sqcap}{\rightsquigarrow} C_1 \sqcap D_2 \rightsquigarrow \dots \rightsquigarrow$$

$$C_1 \sqcap C_2 \rightsquigarrow \dots \rightsquigarrow C_1 \sqcap \dots \sqcap C_m$$

Using this, we can prove completeness (again, we refer to [10] for proofs).

Proposition 3. *$\rho_\downarrow$ is complete.*

Proof. Let C and D be arbitrary $\mathcal{ALC}$ concepts in $S_\downarrow$ with $C \sqsubset_\mathcal{T} D$. To prove completeness of $\rho_\downarrow$ we have to show that there exists a concept E with $E \equiv C$ and $E \in \rho_\downarrow^*(D)$. $E = D \sqcap C$ satisfies this property. We obviously have $E = D \sqcap C \equiv C$, because of $C \sqsubset_\mathcal{T} D$. We know that $\rho_\downarrow$ allows to extend concepts conjunctively by refinements of the top concept. Hence, we know that $D \sqcap C$ can be reached from D for any concept C by the weak completeness result for $\rho_\downarrow$. Thus $\rho_\downarrow$ is complete.

3.2 Achieving Properness

The operator $\rho_\downarrow$ is not proper, for instance it allows the refinement $\top \rightsquigarrow \exists r.\top \sqcup \forall r.A_1 \quad (\equiv \top)$ where $A_1 \in \text{nb}_\downarrow(\top)$. Indeed, there is no structural subsumption algorithm for $\mathcal{ALC}$ [1], which indicates that it is hard to define a proper operator just by syntactic rewriting rules. One could try to modify $\rho_\downarrow$, such that it becomes proper. Unfortunately, this is likely to lead to incompleteness. Say, we disallow the refinement step just mentioned and consider the following refinement chain:

$$\top \rightsquigarrow \exists r.\top \sqcup \forall r.A_1 \rightsquigarrow \exists r.A_2 \sqcup \forall r.A_1 \quad (A_1, A_2 \in \text{nb}_\downarrow(\top))$$

If we disallow the first step, we would have to ensure that we can reach $\exists r.A_2 \sqcup \forall r.A_1$ from $\top$, otherwise the operator is weakly incomplete. In particular, there can be infinite chains of improper refinements:

$$\top \rightsquigarrow \exists r.\top \sqcup \forall r.A_1 \rightsquigarrow \exists r.(\exists r.\top \sqcup \forall r.A_1) \sqcup \forall r.A_1 \rightsquigarrow \dots$$

This example illustrates that one would have to allow very complex concepts to be generated as refinements of the top concept, if one wants to achieve weak completeness and properness. So, instead of modifying $\rho_\downarrow$ directly, we allow it to be improper, but consider the closure $\rho_\downarrow^{cl}$ of $\rho_\downarrow$ [2].

Definition 5 ($\rho_\downarrow^{cl}$). *$\rho_\downarrow^{cl}$ is defined as follows: $D \in \rho_\downarrow^{cl}(C)$ iff there exists a refinement chain*

$$C \rightsquigarrow_{\rho_\downarrow} C_1 \rightsquigarrow_{\rho_\downarrow} \dots \rightsquigarrow_{\rho_\downarrow} C_n = D$$

such that $C \not\equiv_\mathcal{T} D$ and $C_i \equiv C$ for $i \in \{1, \dots, n-1\}$.

$\rho_\downarrow^{cl}$ is proper by definition. It also inherits the weak completeness of $\rho_\downarrow$, since we do not disallow any refinement steps, but only check whether they are improper. However, it is necessary to show that $\rho_\downarrow^{cl}$ is a meaningful operator, which we will do in the sequel. We already know that $\rho_\downarrow$ is infinite, so it is clear that we cannot consider all refinements of a concept at a time. Therefore, in practise we will always compute all refinements of a concept up to a given length. A flexible algorithm will allow this length limit to be increased if necessary. Using this technique, an infinite operator can be handled. However, we have to make sure that all refinements up to a given length are computable in finite time. To show this, we need the following lemma.

Lemma 2 ($\rho_\downarrow$ does not reduce length). *$D \in \rho_\downarrow(C)$ implies $|D| \geq |C|$. Furthermore, there are no infinite refinement chains of the form $C_1 \rightsquigarrow_{\rho_\downarrow} C_2 \rightsquigarrow_{\rho_\downarrow} \ldots$ with $|C_1| = |C_2| = \ldots$, i.e. after a finite number of steps we reach a strictly longer concept.*

Proof. To show the first statement we need to observe the steps, which are performed by $\rho_\downarrow$. We can see that $\rho_\downarrow$ does one of four things in each refinement step:

1. add an element conjunctively ($\overset{\sqcap}{\rightsquigarrow}$)
2. refine the top concept ($\overset{\top}{\rightsquigarrow}$)
3. generalize an atomic concept ($\overset{A}{\rightsquigarrow}$)
4. generalise a negated atomic concept ($\overset{\neg A}{\rightsquigarrow}$)

Steps 1 and 2 obviously result in a concept with greater length. Step 3 and 4 result in a concept with the same length. This proves claim 1.

Claim 2 follows from the fact that there is just a finite number of atomic concepts (N_C is finite) and there are only finitely many occurrences of an atomic concept within any $\mathcal{ALC}$ concept. Hence, there are no infinite refinement chains using only steps 3 or 4. Thus, after a finite number of refinements, step 1 or 2 is used, which produce a longer concept.

Proposition 4 (usefulness of $\rho_\downarrow^{cl}$). *For any concept C in negation normal form and any natural number n, the set $\{D \mid D \in \rho_\downarrow^{cl}(C), |D| \leq n\}$ can be computed in finite time.*

Proof. Due to Lemma 2, we know that for any concept D in the set, there exists an m such that $|D| > |C|$ with $D \in \rho_\downarrow^m(C)$. Obviously, a concept has only finitely many refinements up to a fixed length. If we consider all refinement chains of a concept C by $\rho_\downarrow$ up to length n as a tree, then this tree is finite (there are only finitely many concepts of length $\leq n$ and any such concept can be reached by a finite refinement chain). The set $\{D \mid D \in \rho_\downarrow^{cl}(C), |D| \leq n\}$ is a subset of the nodes of this tree. Hence, it can be computed in finite time.

Due to Proposition 4 we can use $\rho_\downarrow^{cl}$ in a learning algorithm. For computing $\rho_\downarrow^{cl}$ up to n, it is sufficient to apply the operator until a non-equivalent concept is reached. By a straightforward analysis of the refinement steps, one can show that in the worst case after $O(|N_C| \cdot |C|)$ steps a refinement of greater length will be reached, which bounds the complexity of computing the closure.

4 The Learning Algorithm

So far, we have designed a complete and proper operator. Unfortunately, such an operator has to be redundant and infinite by Theorem 1. We will now describe how to deal with these problems and define the overall learning algorithm.

4.1 Redundancy Elimination

A learning algorithm can be constructed as a combination of a refinement operator, which defines how the search tree can be built, and a search algorithm, which controls how the tree is traversed. The search algorithm specifies which nodes have to be expanded. Whenever the search algorithm encounters a node in the search tree, it could check whether a weakly equal concept already exists in the search tree. If yes, then this node is ignored, i.e. it will not be expanded further and it will not be evaluated. This removes all redundancies, since every concept exists at most once in the search tree.[4] We can still reach any concept, because we have $\rho_\downarrow^{cl}(C) \simeq \rho_\downarrow^{cl}(D)$ if $C \simeq D$, i.e. $\rho_\downarrow^{cl}$ handles weakly equal concepts in the same way. However, this redundancy elimination approach is computationally expensive if performed naively. Hence, we considered it worthwhile to investigate how it can be handled as efficiently as possible.

Note, that we consider weak equality instead of equality here, e.g. we have $A_1 \sqcap A_2 \neq A_2 \sqcap A_1$, but $A_1 \sqcap A_2 \simeq A_2 \sqcap A_1$. In conjunctions and disjunctions, we have the problem that we have to guess which pairs of elements are equal to determine whether two concepts are weakly equal. One way to solve this problem is to define an ordering over concepts and require the elements of disjunctions and conjunctions to be ordered accordingly. This eliminates the guessing step and allows to check weak equality in linear time. There are different ways to define a linear order $\preceq$ over $\mathcal{ALC}$ concepts, and we have shown that it is also possible to do it in such a way that deciding $\preceq$ for two concepts is polynomial and transforming a concept in negation normal form to $\preceq$ *ordered negation normal form*, i.e. elements in conjunctions and disjunctions are ordered with respect to $\preceq$, can be done in polynomial time – for brevity we omit the details. It is thus reasonable to assume that every concept occurring in our search tree can be transformed to ordered negation normal form with respect to some linear order over $\mathcal{ALC}$ concepts. We can then maintain an ordered set of concepts occurring in the search tree. Checking weak equality of a concept C with respect to a search tree containing n concepts will then only require $\log n$ comparisons (binary search), where each comparison needs only linear time. Taking into account the complexity of instance checks (PSPACE for $\mathcal{ALC}$, NEXPTIME for $\mathcal{SHOIN}(D)$ and

[4] More precisely: For each concept there is at most one representative of the equivalence class of weakly syntactical equal concepts in the search tree which is evaluated.

OWL-DL), which we can avoid (compared to an algorithm without redundancy check), redundancy elimination can be considered reasonable.

4.2 Creating a Full Learning Algorithm

Learning concepts in DLs is a search process. In our proposed learning algorithm, the refinement operator $\rho_{\downarrow}^{cl}$ is used for building the search tree, while a heuristics decides which nodes to expand. To define a search heuristics for our learning algorithm, we need some notions to be able to express what we consider a good concept.

Definition 6 (quality). *Let $\mathcal{K}$ be a knowledge base, E^- the set of negative examples, and E^+ the set of positive examples of a learning problem. The* quality *of a concept C is a function, which maps a concept to an element of $\mathcal{Q}$ with $\mathcal{Q} = \{0, \ldots, -|E^-|\} \cup \{tw\}$, defined by $q(C) = tw$ if there is an $e \in E^+$ with $\mathcal{K} \cup \{C\} \not\models e$ and $q(C) = -|\{e \mid e \in E^- \text{and } \mathcal{K} \cup \{C\} \models e\}|$ otherwise.*

The quality of a concept is "tw" if it is too weak, i.e. it does not cover all positive examples. In all other cases, we assign a number n with $n \geq 0$ to a concept, which is the number of negative examples covered.

As mentioned before, we want to tackle the infinity of the operator by considering only refinements up to some length n at a given time. We call n the *horizontal expansion* of a node. It is a node specific upper bound on the length of child concepts, which can be increased dynamically by the algorithm during the learning process. To deal with this, we formally define a *node* in a search tree to be a quadruple (C, n, q, b), where C is an $\mathcal{ALC}$ concept, $n \in \mathbb{N}$ is the *horizontal expansion*, $q \in \mathcal{Q} \cup \{\text{-}\}$ is the *quality* (- stands for non-evaluated quality), and $b \in \{\text{true}, \text{false}\}$ is a boolean marker for the redundancy of a node.

The search heuristics selects the fittest node in the search tree at a given time. We define fitness as a lexicographical order over quality and horizontal expansion.

Definition 7 (fitness). *Let $N_1 = (C_1, n_1, q_1, b_1)$ and $N_2 = (C_2, n_2, q_2, b_2)$ be two nodes with defined quality ($q_1, q_2 \neq -, tw$). N_1 is* fitter *than N_2, denoted by $N_2 \leq_f N_1$ iff $q_2 < q_1$ or $q_1 = q_2$ and $n_1 \leq n_2$.*

Note, that we use horizontal expansion instead of concept length as second criterion, which makes the algorithm more flexible in searching less explored areas of the search space. More sophisticated ways of ordering concepts are also possible, e.g. tradeoffs between quality and horizontal expansion. Such fitness heuristics enable the algorithm to handle noise. The fitness function can be defined independently of the core learing algorithm.

We have now introduced all necessary notions to specify the complete learning algorithm, given in Algorithm 1. *checkRed* is the redundancy check function and *transform* the function to transform a concept to ordered negation normal form.

We see, that the usual expansion in a search algorithm is replaced by a one step horizontal expansion. If we only expand the fittest node, we may not explore

Algorithm 1: learning algorithm

Input: *horizExpFactor* in]0,1]

ST (search tree) is set to the tree consisting only of the root node $(\top, 0, q(\top), \mathit{false})$

$\mathit{minHorizExp} = 0$

while *ST does not contain a correct concept* **do**

  choose $N = (C, n, q, b)$ with highest fitness in *ST*

  expand N up to length $n + 1$, i.e. :

  begin

    add all nodes $(D, n, -, \mathit{checkRed}(ST, D))$ with $D \in \mathit{transform}(\rho_{\downarrow}^{cl}(C))$ and $|D| = n + 1$ as children of N

    evaluate created non-redundant nodes

    change N to $(C, n + 1, q, b)$

  end

  $\mathit{minHorizExp} = \max(\mathit{minHorizExp}, \lceil \mathit{horizExpFactor} * (n + 1) \rceil)$

  while *there are nodes with defined quality and horiz. expansion smaller minHorizExp* **do**

    expand these nodes up to *minHorizExp*

Return a correct concept in *ST*

large parts of the search space. In order to avoid this, a minimum horizontal expansion factor is used, which specifies that all nodes have to be expanded at least up to this length. This factor allows us to control the tradeoff between expanding only the fittest nodes and exploring other parts of the search space.

Correctness of the algorithm can be shown:

Proposition 5 (correctness). *If a learning problem has a solution, then Algorithm 1 terminates and computes a correct solution of the learning problem.*

Proof. Assume, there is a solution C (which is an $\mathcal{ALC}$ concept) of a learning problem. By the weak completeness of $\rho_{\downarrow}^{cl}$, we know that there is a concept D with $D \equiv C$ and $D \in \rho_{\downarrow}^{*}(\top)$. Because the horizontal expansion factor in Algorithm 1 is higher than 0, we will eventually explore D unless another solution is found before. In both cases the proposition is satisfied.

5 Preliminary Evaluation

We want to illustrate our algorithm using Michalski's trains [12] as an example. The data describes different features of trains, e.g. which cars are appended to a train, whether they are short or long, closed or open, jagged or not, which shapes they contain and how many of them. The positive examples are the trains on the left and the negative examples are the trains on the right. Thus, the task of the learner is to find characteristics of all the left trains, which none of the right trains has. The learning algorithm first explores the concepts $\top$ and `Train`, which cover all examples. Other atomic concepts are too weak to be considered for further

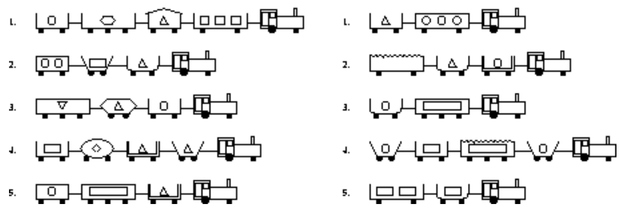

Figure 2. Michalski trains

exploration. The exploration of the top concept leads to $\exists$`hasCar`.$\top$, which is then expanded to $\exists$`hasCar.Closed`. This covers all positives and two negatives. The heuristic picks this node and extends it to $\exists$`hasCar`.(`Closed`$\sqcap$`Short`), which is a possible (and shortest) solution for the problem.

Doubtless, there is a lack of evaluation standards in ontology learning from examples. In order to overcome this problem, we converted the background knowledge of several existing learning problems to OWL ontologies. Besides the described train problem, we also investigated the problems of learning arches [14], learning poker hands, and understanding the moral reasoning of humans. The two latter examples were taken from the UCI Machine Learning repository[5]. For the poker example, we defined two goals: learning the definition of a pair and of a straight. Similarly, the moral reasoner examples were divided into two learning tasks: the original one, where the intended solution is quite short, and a problem, where we removed an important intermediate concept, such that the smallest possible solution became more complex.

The arch problem is small in terms of size and complexity of the background knowledge. The poker example is larger in terms of size, but still not very complex. The moral reasoner, however, is an expressive ontology, which we derived from a theory given as logic program. The solutions of the examples cover a range of different concept constructors and are of varying length and complexity.

For all test runs we used a (non-optimised) horizontal expansion factor of 0.6. As a reasoner we used Pellet[6] (version 1.4RC1), which was connected to the learning program using the DIG 1.1 interface[7] on a 1.4 GHz CPU machine. The only system we could use for comparison is YinYang [8]. The system in [5] is no longer available and the approach in [2] was not fully implemented.

Table 2 summarises the results we obtained. In all cases, our implementation – called DL-Learner – was able to learn the shortest correct definition (which coincides with the intended solution of these problems). YinYang produces longer solutions and could not solve the second poker problem (it produces an error after trying to compute most specific concepts for some time). It generated an incorrect answer for both moral reasoner problems. The percentage of time

[5] http://www.ics.uci.edu/~mlearn/MLRepository.html
[6] http://pellet.owldl.com
[7] http://dl.kr.org/dig/

spent for reasoner requests increases with the complexity and size of background knowledge and the number of examples (it is $> 99\%$ for the moral reasoner problems), which shows that minimizing the number of reasoner requests, e.g. by redundancy elimination, is an important issue.

problem	axioms, concepts, roles objects, examples	DL-Learner			YinYang		
		runtime	length	correct	runtime	length	correct
trains	252, 8, 5, 50, 10	1.1s	5	100%	2.3s	8	100%
arches	71, 6, 5, 19, 5	4.6s	9	100%	1.5s	23	100%
moral (simple)	2176, 43, 4, 45, 43	17.7s	3	100%	205.3s	69	67.4%
moral (complex)	2107, 40, 4, 45, 43	88.1s	8	100%	181.4s	70	69.8%
poker (pair)	1335, 2, 6, 311, 49	7.7s	5	100%	17.1s	43	100%
poker (straight)	1419, 2, 6, 347, 55	35.6s	11	100%	-	-	-

Table 2. evaluation results for DL-Learner and YinYang

6 Related Work

An interesting paper close to our work is [2]. It suggests a refinement operator for the $\mathcal{ALER}$ description logic. They also investigate some theoretical properties of refinement operators. As we have done with the design of $\rho_\downarrow$, they favour the use of a downward refinement operator to enable a top-down search. The authors use $\mathcal{ALER}$ *normal form* (see the paper for a detailed description), which is easier to handle than negation normal form, because $\mathcal{ALER}$ is not closed under boolean operations. As a consequence, they obtain a simpler refinement operator, for which it is not clear how it could be extended to more expressive DLs. Our operator, in contrast, lends itself much easier to such extensions. We also deal quite differently with infinity, show how the subsumption hierarchy of atomic concepts can be used, and describe how redundancy can be avoided efficiently.

A second area of ongoing related work is described in [6,7,8]. They take a different approach for solving the learning problem by using approximated MSCs (most specific concepts). A problem of these algorithms is that they tend to produce unnecessarily long concepts. One reason is that MSCs for $\mathcal{ALC}$ and more expressive languages do not exist and hence can only be approximated. Previous work [4,5] in learning in DLs has mostly focused on approaches using least common subsumers, which face this problem to an even larger extent.

In our approach, we also cannot guarantee that we obtain the shortest possible solution of a learning problem. However, the learning algorithm was carefully designed to produce short solutions. The produced solutions will be close to the shortest solution in negation normal form and, thus, overfitting is unlikely.

Another related area of research are approaches for learning in the hybrid language AL-log [11], which combines $\mathcal{ALC}$ with the function free Horn clause language Datalog.

7 Conclusions and Further Work

To the best of our knowledge, our work presents the first refinement operator based learning algorithm for expressive DLs which are closed under boolean operations.[8] It is based on thorough theoretical investigations concerning the potential of using refinement operators for DLs, and we have shown formally that our operator satisfies the desirable properties which are achievable. We also showed how the problems of redundancy and infinity can be solved in a satisfiable manner, allowing us to specify a learning algorithm which we proved to be correct. We implemented the algorithm and an evaluation showed the feasibility of our approach.

Future work will focus on increasing the expressiveness of the learned language, integrating the learning algorithm in an ontology editor, creating benchmark datasets, and testing on real world data sets.

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[8] To be precise, [8] also uses refinement operators, but not as centrally as in our approach – see Section 6.

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