$\mathcal{ALC}$ Concept Learning with Refinement Operators

Jens Lehmann

Artificial Intelligence Institute
Computer Science Department
Technische Universität Dresden

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Outline

1. the learning problem
2. refinement operators and their properties
3. the refinement operators $\rho_\downarrow$ and $\rho_\downarrow^{cl}$
4. conclusions and future work

Learning Problem

- goal: learn a concept definition from positive examples + negative examples + background knowledge
- we have a target concept name `Target` and a knowledge base $\mathcal{K}$ as background knowledge
- examples are of the form $\texttt{Target}(a)$, where a is an object
- let E^+ be the set of positive examples and E^- the set of negative examples
- we want to find a definition *Def* of the form $\texttt{Target} \equiv C$ such that for $\mathcal{K}' = \mathcal{K} \cup \{Def\}$ we have $\mathcal{K}' \models E^+$ and $\mathcal{K}' \not\models E^-$

Simple Example

$\texttt{Male} \equiv \neg\texttt{Female}$	$\texttt{Male}(\texttt{MARC})$
	$\texttt{Male}(\texttt{STEPHEN})$
$\texttt{hasChild}(\texttt{STEPHEN},\texttt{MARC})$	$\texttt{Male}(\texttt{JASON})$
$\texttt{hasChild}(\texttt{MARC},\texttt{ANNA})$	$\texttt{Male}(\texttt{JOHN})$
$\texttt{hasChild}(\texttt{JOHN},\texttt{MARIA})$	$\texttt{Female}(\texttt{ANNA})$
$\texttt{hasChild}(\texttt{ANNA},\texttt{JASON})$	$\texttt{Female}(\texttt{MARIA})$
	$\texttt{Female}(\texttt{MICHELLE})$

positive: $\{\texttt{STEPHEN}, \texttt{MARC}, \texttt{JOHN}\}$

negative: $\{\texttt{JASON}, \texttt{ANNA}, \texttt{MARIA}, \texttt{MICHELLE}\}$

learned concept: $\texttt{Male} \sqcap \exists\texttt{hasChild}.\top$

Application Areas

Why is it useful to learn in DLs?

- may have similar applications like ILP (Inductive Logic Programming) approaches for learning horn clauses e.g. in biology and medicine
- incremental ontology learning in context of OWL and the Semantic Web – the lack of ontologies is a bottleneck in the Semantic Web

Solving the Learning Problem

Two learning approaches in my thesis:

1. usage of refinement operators combined with a search heuristic (main topic of presentation)
    - search in the space of concepts ordered by subsumption
    - similar to traditional Inductive Logic Programming methods
2. usage of Genetic Programming
    - has been used (with non-standard operators) in ILP to induce logic programs
    - in thesis, insights about refinement operators were used to improve standard GP, resulting in a hybrid approach
    - proposed extensions: learning from uncertain examples, concept invention

Refinement Operators - Definitions

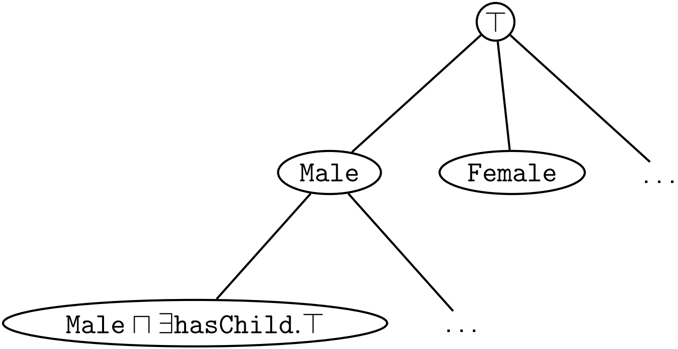
- consider quasi-ordered space $(S, \preceq)$, i.e. $\preceq$ is reflexive and transitive
- downward (upward) refinement operator ρ is a mapping from S to 2^S such that for any $C \in S$:

$$C' \in \rho(C) \text{ implies } C' \preceq C \quad (C \preceq C')$$

- refinement operator in the quasi-ordered space $(\mathcal{ALC}, \sqsubseteq_{\mathcal{T}})$ is called an $\mathcal{ALC}$ refinement operator
- a refinement chain of an $\mathcal{ALC}$ refinement operator ρ from a concept C to a concept D is a finite sequence $C_0, C_1, \ldots, C_n$ of concepts, such that
  $C = C_0, C_1 \in \rho(C_0), C_2 \in \rho(C_1), \ldots, C_n \in \rho(C_{n-1}), D = C_n$
- instead of $D \in \rho(C)$ we often write $C \rightsquigarrow_\rho D$, e.g.
  $\top \rightsquigarrow_\rho \texttt{Male} \rightsquigarrow_\rho \texttt{Male} \sqcap \exists \texttt{hasChild}.\top$

Learning with Refinement Operators

- refinement operator can be used to span up a search tree
- refinement operator + search heuristic = learning algorithm



Properties of Refinement Operators

An $\mathcal{ALC}$ refinement operator ρ is called

- finite iff $\rho(C)$ is finite for any concept C.
- redundant iff there exist two different refinement chains from a concept C to a concept D.
- proper iff for any concepts C and D, $D \in \rho(C)$ implies $C \not\equiv_{\mathcal{T}} D$.

An $\mathcal{ALC}$ downward refinement operator is called

- complete iff for any concepts C and D with $C \sqsubset_{\mathcal{T}} D$ we can reach a concept E with $E \equiv_{\mathcal{T}} C$ from D by ρ.
- weakly complete iff for any concept C with $C \sqsubset_{\mathcal{T}} \top$ we can reach a concept E with $E \equiv_{\mathcal{T}} C$ from $\top$ by ρ.

Theorem about Properties of $\mathcal{ALC}$ Refinement Operators

Theorem (properties of $\mathcal{ALC}$ refinement operators)

Considering these properties, the following are maximal sets of properties of $\mathcal{ALC}$ refinement operators:

1. {*weakly complete*, *complete*, *finite*}
2. {*weakly complete*, *complete*, *proper*}
3. {*weakly complete*, *non-redundant*, *finite*}
4. {*weakly complete*, *non-redundant*, *proper*}
5. {*non-redundant*, *finite*, *proper*}

$\rho_\downarrow$ part 1 of 2

$$\rho_\downarrow(C) = \begin{cases} \{\bot\} \cup \rho'_\downarrow(C) & \text{if } C = \top \\ \rho'_\downarrow(C) & \text{otherwise} \end{cases}$$

$$\rho'_\downarrow(C) = \begin{cases} \{C_1 \sqcap \dots \sqcap C_{i-1} \sqcap D \sqcap C_{i+1} \sqcap \dots \sqcap C_n & \text{if } C = C_1 \sqcap \dots \sqcap C_n \\ \quad \mid D \in \rho'_\downarrow(C_i), 1 \leq i \leq n\} & \\ \{C_1 \sqcup \dots \sqcup C_{i-1} \sqcup D \sqcup C_{i+1} \sqcup \dots \sqcup C_n & \text{if } C = C_1 \sqcup \dots \sqcup C_n \\ \quad \mid D \in \rho'_\downarrow(C_i), 1 \leq i \leq n\} & \\ \cup \{C \sqcap D \mid D \in \rho'_\downarrow(\top)\} & \\ \{A' \mid A' \sqsubset_{\mathcal{T}} A, A' \in N_C, \text{ there is} & \text{if } C = A \, (A \in N_C) \\ \text{ no } A'' \in N_C \text{ with } A' \sqsubset_{\mathcal{T}} A'' \sqsubset_{\mathcal{T}} A\} & \\ \cup \{C \sqcap D \mid D \in \rho'_\downarrow(\top)\} & \\ \{\neg A' \mid A \sqsubset_{\mathcal{T}} A', A' \in N_C, \text{ there is} & \text{if } C = \neg A \, (A \in N_C) \\ \text{ no } A'' \in N_C \text{ with } A \sqsubset_{\mathcal{T}} A'' \sqsubset_{\mathcal{T}} A'\} & \\ \cup \{C \sqcap D \mid D \in \rho'_\downarrow(\top)\} & \end{cases}$$

$\rho_\downarrow$ part 2 of 2

$$\rho'_\downarrow(C) = \begin{cases} \{\exists r.E \mid E \in \rho'_\downarrow(D)\} & \text{if } C = \exists r.D \\ \cup \{C \sqcap D \mid D \in \rho'_\downarrow(\top)\} & \\ \{\forall r.E \mid E \in \rho'_\downarrow(D)\} \cup \{C \sqcap D \mid D \in \rho'_\downarrow(\top)\} & \text{if } C = \forall r.D \\ \cup \{\forall r.\bot \mid D = A \in N_C & \\ \text{ and there is no } A' \in N_C \text{ with } \bot \sqsubset A' \sqsubset A\} & \\ \emptyset & \text{if } C = \bot \\ \{D \mid D \in M\} & \text{if } C = \top \\ \cup \{D \sqcup E \mid D \in M, E \in \rho'_\downarrow(\top)\} & \\ M = & \\ \{A \mid A \in N_C, & \\ \quad \text{there is no } A' \in N_C \text{ with } A \sqsubset_{\mathcal{T}} A' \sqsubset_{\mathcal{T}} \top\} & \\ \cup \{\neg A \mid A \in N_C, & \\ \quad \text{there is no } A' \in N_C \text{ with } \bot \sqsubset_{\mathcal{T}} A' \sqsubset_{\mathcal{T}} A\} & \\ \cup \{\exists r.\top \mid r \in N_R\} & \\ \cup \{\forall r.C \mid r \in N_R, C \in \rho'_\downarrow(\top)\} & \end{cases}$$

Completeness of $\rho_\downarrow$

Proposition (completeness of $\rho_\downarrow$)

$\rho_\downarrow$ is complete.

Proof Idea:

- first show weak completeness:
  - a set $S_\downarrow$ of $\mathcal{ALC}$ concepts was defined (see thesis for the definition of $S_\downarrow$)
  - for every $\mathcal{ALC}$ concept there exists an equivalent concept in $S_\downarrow$
  - all concepts in $S_\downarrow$ can be reached by $\rho_\downarrow$ from $\top$
- prove completeness using the weak completeness result

Infiniteness of $\rho_\downarrow$

- $\rho_\downarrow$ is infinite, e.g. there are infinitely many refinement steps of the form:

$$\top \leadsto_{\rho_\downarrow} \underbrace{\forall \texttt{hasChild}.\ldots.\forall \texttt{hasChild}}_{\text{arbitrarily often}}.\texttt{Male}$$

- solution: we only consider refinements up to length n of concepts (there are only finitely many of these)
- n is initially set to 0 and increased by the learning algorithm as needed

Properness

- $\rho_\downarrow$ is not proper: $\top \rightsquigarrow_{\rho_\downarrow} \exists\mathtt{hasChild}.\top \sqcup \forall\mathtt{hasChild}.\mathtt{Male}$
- idea: consider the closure $\rho_\downarrow^{cl}$ of $\rho_\downarrow$:
  $D \in \rho_\downarrow^{cl}(C)$ iff there exists a refinement chain

$$C \rightsquigarrow_{\rho_\downarrow} C_1 \rightsquigarrow_{\rho_\downarrow} \ldots \rightsquigarrow_{\rho_\downarrow} C_n = D$$

  such that $C \not\equiv D$ and $C_i \equiv C$ for $i \in \{1, \ldots, n-1\}$

Proposition

For any concept C in negation normal form and any natural number n the set

$$\{D \mid D \in \rho_\downarrow^{cl}(C), |D| \leq n\}$$

can be computed in finite time.

Redundancy

$\rho_{\downarrow}^{cl}$ is redundant:

$$\begin{tikzcd}
& \forall r_1.A_1 \sqcup \forall r_2.A_1 \arrow[r, rightsquigarrow, "\rho_{\downarrow}"'] \arrow[d, rightsquigarrow, "\rho_{\downarrow}"'] & \forall r_1.(A_1 \sqcap A_2) \sqcup \forall r_2.A_1 \arrow[d, rightsquigarrow, "\rho_{\downarrow}"'] \\
& \forall r_1.A_1 \sqcup \forall r_2.(A_1 \sqcap A_2) \arrow[r, rightsquigarrow, "\rho_{\downarrow}"'] & \forall r_1.(A_1 \sqcap A_2) \sqcup \forall r_2.(A_1 \sqcap A_2)
\end{tikzcd}$$

- redundancies should be detected by the learning algorithm
- result in thesis: we can check whether an occurring concept is redundant with respect to a search tree in polynomial time

Conclusions

- first full analysis of theoretical properties of refinement operators for Description Logics
- complete, proper operator was defined and ways to handle infinity and redundancy were shown
- theoretical results ensure that the learning algorithm is close to the best we can hope for
- result: if a solution exists, then the algorithm terminates in finite time and finds a solution

Related Work (Excerpt)

Badea, L. and Nienhuys-Cheng, S.-H. (2000). A refinement operator for description logics. *Proceedings of the 10th International Conference on Inductive Logic Programming*

Esposito, F., Fanizzi, N., Iannone, L., Palmisano, I., and Semeraro, G. (2004). Knowledge-intensive induction of terminologies from metadata. *Proceedings of the Third International Semantic Web Conference*

Iannone, L. and Palmisano, I. (2005). An algorithm based on counterfactuals for concept learning in the semantic web. *Proceedings of the 18th International Conference on Industrial and Engineering Applications of Artificial Intelligence and Expert Systems*

Future Work

- implement learning algorithms (refinement operator based approach with a search heuristic, Genetic Programming approach)
- create benchmarks
- embed learning algorithm in ontology editor
- extend theory to other description languages and OWL

Thank you for your attention.