ALC Concept Learning with Refinement Operators

Jens Lehmann

Artificial Intelligence Institute
Computer Science Department
Technische Universität Dresden

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Outline

1. the learning problem
2. refinement operators and their properties
3. the refinement operators $\rho_\downarrow$ and $\rho_{cl}^{\downarrow}$
4. conclusions and future work
Learning Problem

- goal: learn a concept definition from positive examples + negative examples + background knowledge
- we have a target concept name \( \text{Target} \) and a knowledge base \( \mathcal{K} \) as background knowledge
- examples are of the form \( \text{Target}(a) \), where \( a \) is an object
- let \( E^+ \) be the set of positive examples and \( E^- \) the set of negative examples
- we want to find a definition \( \text{Def} \) of the form \( \text{Target} \equiv C \) such that for \( \mathcal{K}' = \mathcal{K} \cup \{\text{Def}\} \) we have \( \mathcal{K}' \models E^+ \) and \( \mathcal{K}' \not\models E^- \)
Simple Example

<table>
<thead>
<tr>
<th>Male ≡ ¬Female</th>
<th>Male(MARC)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Male(STEPHEN)</td>
</tr>
<tr>
<td>hasChild(STEPHEN,MARC)</td>
<td>Male(JASON)</td>
</tr>
<tr>
<td>hasChild(MARC,ANNA)</td>
<td>Male(JOHN)</td>
</tr>
<tr>
<td>hasChild(JOHN,MARIA)</td>
<td>Female(ANNA)</td>
</tr>
<tr>
<td>hasChild(ANNA,JASON)</td>
<td>Female(MARIA)</td>
</tr>
<tr>
<td></td>
<td>Female(MICHELLE)</td>
</tr>
</tbody>
</table>

positive: {STEPHEN, MARC, JOHN}

negative: {JASON, ANNA, MARIA, MICHELLE}

learned concept: Male ∩ ∃ hasChild. ⊤
Application Areas

Why is it useful to learn in DLs?

- may have similar applications like ILP (Inductive Logic Programming) approaches for learning horn clauses e.g. in biology and medicine

- incremental ontology learning in context of OWL and the Semantic Web – the lack of ontologies is a bottleneck in the Semantic Web
Solving the Learning Problem

Two learning approaches in my thesis:

1. usage of refinement operators combined with a search heuristic (main topic of presentation)
   - search in the space of concepts ordered by subsumption
   - similar to traditional Inductive Logic Programming methods

2. usage of Genetic Programming
   - has been used (with non-standard operators) in ILP to induce logic programs
   - in thesis, insights about refinement operators were used to improve standard GP, resulting in a hybrid approach
   - proposed extensions: learning from uncertain examples, concept invention
Refinement Operators - Definitions

- consider quasi-ordered space \((S, \preceq)\), i.e. \(\preceq\) is reflexive and transitive
- downward (upward) refinement operator \(\rho\) is a mapping from \(S\) to \(2^S\) such that for any \(C \in S\):
  \[
  C' \in \rho(C) \text{ implies } C' \preceq C \quad (C \preceq C')
  \]
- refinement operator in the quasi-ordered space \((\mathcal{ALC}, \sqsubseteq_T)\) is called an \(\mathcal{ALC}\) refinement operator
- a refinement chain of an \(\mathcal{ALC}\) refinement operator \(\rho\) from a concept \(C\) to a concept \(D\) is a finite sequence \(C_0, C_1, \ldots, C_n\) of concepts, such that
  \[
  C = C_0, C_1 \in \rho(C_0), C_2 \in \rho(C_1), \ldots, C_n \in \rho(C_{n-1}), D = C_n
  \]
- instead of \(D \in \rho(C)\) we often write \(C \rightsquigarrow \rho D\), e.g.
  \[
  \top \rightsquigarrow \rho \text{Male} \rightsquigarrow \rho \text{Male} \sqcap \exists \text{hasChild. T}
  \]
Learning with Refinement Operators

- Refinement operator can be used to span up a search tree
- Refinement operator + search heuristic = learning algorithm

Diagram:

- Male
- Female
- Male ∧ ∃ hasChild. ⊤
Properties of Refinement Operators

An $\mathcal{ALC}$ refinement operator $\rho$ is called

- **finite** iff $\rho(C)$ is finite for any concept $C$.
- **redundant** iff there exist two different refinement chains from a concept $C$ to a concept $D$.
- **proper** iff for any concepts $C$ and $D$, $D \in \rho(C)$ implies $C \not\equiv_T D$.

An $\mathcal{ALC}$ downward refinement operator is called

- **complete** iff for any concepts $C$ and $D$ with $C \sqsubseteq_T D$ we can reach a concept $E$ with $E \equiv_T C$ from $D$ by $\rho$.
- **weakly complete** iff for any concept $C$ with $C \sqsubseteq_T \top$ we can reach a concept $E$ with $E \equiv_T C$ from $\top$ by $\rho$. 
Theorem (properties of $\mathcal{ALC}$ refinement operators)

Considering these properties, the following are maximal sets of properties of $\mathcal{ALC}$ refinement operators:

1. $\{\text{weakly complete, complete, finite}\}$
2. $\{\text{weakly complete, complete, proper}\}$
3. $\{\text{weakly complete, non-redundant, finite}\}$
4. $\{\text{weakly complete, non-redundant, proper}\}$
5. $\{\text{non-redundant, finite, proper}\}$
\[\rho \downarrow \text{ part 1 of 2}\]

\[
\rho_\downarrow(C) = \begin{cases} 
\{\bot\} \cup \rho'_\downarrow(C) & \text{if } C = \top \\
\rho'_\downarrow(C) & \text{otherwise}
\end{cases}
\]

\[
\rho'_\downarrow(C) = \begin{cases} 
\{C_1 \cap \cdots \cap C_{i-1} \cap D \cap C_{i+1} \cap \cdots \cap C_n \ |
D \in \rho'_\downarrow(C_i), 1 \leq i \leq n\} & \text{if } C = C_1 \cap \cdots \cap C_n \\
\{C_1 \cup \cdots \cup C_{i-1} \cup D \cup C_{i+1} \cup \cdots \cup C_n \ |
D \in \rho'_\downarrow(C_i), 1 \leq i \leq n\} & \text{if } C = C_1 \cup \cdots \cup C_n \\
\{A' \mid A' \sqcap_T A, A' \in N_C, \text{ there is no } A'' \in N_C \text{ with } A' \sqcap_T A'' \sqcap_T A\} & \text{if } C = A (A \in N_C) \\
\cup \{C \cap D \mid D \in \rho'_\downarrow(\top)\}\}
\end{cases}
\]
\( \rho \downarrow \) part 2 of 2

\[
\rho'_\downarrow (C) = \begin{cases} 
\{ \exists r. E \mid E \in \rho'_\downarrow (D) \} & \text{if } C = \exists r. D \\
\cup \{ C \cap D \mid D \in \rho'_\downarrow (\top) \} & \\
\{ \forall r. E \mid E \in \rho'_\downarrow (D) \} \cup \{ C \cap D \mid D \in \rho'_\downarrow (\top) \} & \text{if } C = \forall r. D \\
\cup \{ \forall r. \bot \mid D = A \in N_C \} & \\
\text{and there is no } A' \in N_C \text{ with } \bot \sqsubseteq A' \sqsubseteq A \} & \\
\emptyset & \text{if } C = \bot \\
\{ D \mid D \in M \} & \text{if } C = \top \\
\cup \{ D \sqcup E \mid D \in M, E \in \rho'_\downarrow (\top) \} \end{cases}
\]

\[
M = \begin{cases} 
\{ A \mid A \in N_C, \text{ there is no } A' \in N_C \text{ with } A \sqsubseteq \top A' \sqsubseteq \top \} & \\
\cup \{ \neg A \mid A \in N_C, \text{ there is no } A' \in N_C \text{ with } \bot \sqsubseteq \top A' \sqsubseteq \top \} & \\
\cup \{ \exists r. T \mid r \in N_R \} & \\
\cup \{ \forall r. C \mid r \in N_R, C \in \rho'_\downarrow (\top) \} \end{cases}
\]
Completeness of $\rho_\downarrow$

**Proposition (completeness of $\rho_\downarrow$)**

$\rho_\downarrow$ is complete.

**Proof Idea:**

- first show weak completeness:
  - a set $S_\downarrow$ of $\mathcal{ALC}$ concepts was defined (see thesis for the definition of $S_\downarrow$)
  - for every $\mathcal{ALC}$ concept there exists an equivalent concept in $S_\downarrow$
  - all concepts in $S_\downarrow$ can be reached by $\rho_\downarrow$ from $\top$

- prove completeness using the weak completeness result
Infiniteness of $\rho_{\downarrow}$

- $\rho_{\downarrow}$ is infinite, e.g. there are infinitely many refinement steps of the form:

$$\top \xrightarrow{\rho_{\downarrow}} \forall \text{hasChild} . . . \forall \text{hasChild} . \text{Male}$$

- solution: we only consider refinements up to length $n$ of concepts (there are only finitely many of these)

- $n$ is initially set to 0 and increased by the learning algorithm as needed
Properness

- $\rho_\downarrow$ is not proper: $\top \rightsquigarrow \rho_\downarrow \exists \text{hasChild.}\top \sqcup \forall \text{hasChild.Male}

- idea: consider the closure $\rho^{\text{cl}}_\downarrow$ of $\rho_\downarrow$:
  
  $D \in \rho^{\text{cl}}_\downarrow(C)$ iff there exists a refinement chain
  
  $$C \rightsquigarrow \rho_\downarrow C_1 \rightsquigarrow \rho_\downarrow \ldots \rightsquigarrow \rho_\downarrow C_n = D$$

  such that $C \not\equiv D$ and $C_i \equiv C$ for $i \in \{1, \ldots, n - 1\}$

Proposition

For any concept $C$ in negation normal form and any natural number $n$ the set

$$\{ D \mid D \in \rho^{\text{cl}}_\downarrow(C), |D| \leq n \}$$

can be computed in finite time.
\( \rho_{\downarrow}^{cl} \) is redundant:

\[
\forall r_1.A_1 \sqcup \forall r_2.A_1 \sim_{\rho_{\downarrow}} \forall r_1.(A_1 \sqcap A_2) \sqcup \forall r_2.A_1
\]

\[
\uparrow_{\rho_{\downarrow}} \quad \uparrow_{\rho_{\downarrow}}
\]

\[
\forall r_1.A_1 \sqcup \forall r_2.(A_1 \sqcap A_2) \sim_{\rho_{\downarrow}} \forall r_1.(A_1 \sqcap A_2) \sqcup \forall r_2.(A_1 \sqcap A_2)
\]

- redundancies should be detected by the learning algorithm
- result in thesis: we can check whether an occurring concept is redundant with respect to a search tree in polynomial time
Conclusions

- first full analysis of theoretical properties of refinement operators for Description Logics
- complete, proper operator was defined and ways to handle infinity and redundancy were shown
- theoretical results ensure that the learning algorithm is close to the best we can hope for
- result: if a solution exists, then the algorithm terminates in finite time and finds a solution


Future Work

- implement learning algorithms (refinement operator based approach with a search heuristic, Genetic Programming approach)
- create benchmarks
- embed learning algorithm in ontology editor
- extend theory to other description languages and OWL
Thank you for your attention.